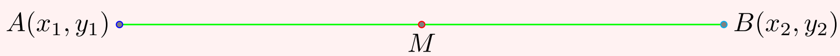
LESSON 7: Midpoint of a line segment

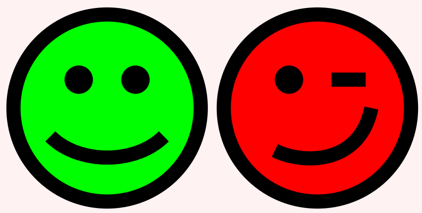
The midpoint formula

✍ if a line segment AB has endpoints at $A(x_1, y_1)$ and $B(x_2, y_2)$:



then its midpoint M is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

✍ Easy-peasy.



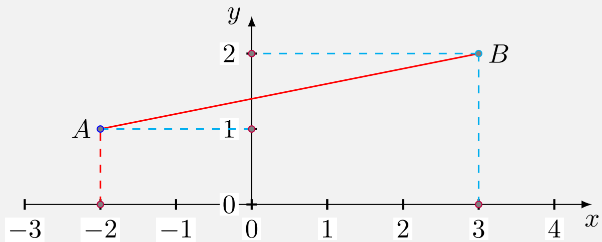
EXAMPLE 1: Find the midpoint of $A(1, 4)$ and $B(9, 2)$.

We use the above midpoint formula with $A(x_1, y_1) = A(1, 4)$ and $B(x_2, y_2) = B(9, 2)$. Basically, $x_1 = 1,\ y_1 = 4$ and $x_2 = 9,\ y_2 = 2$. So:

$$\begin{aligned} \textit{midpoint} &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{1+9}{2}, \frac{4+2}{2}\right) \\ &= \left(\frac{10}{2}, \frac{6}{2}\right) \\ &= (5, 3) \end{aligned}$$

The midpoint is at $(5, 3)$.

EXAMPLE 2: Find the midpoint of the line segment AB in the diagram below:



From the diagram, A is the point $(-2, 1)$, while B is the point $(3, 2)$. Put $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 2)$ in the midpoint formula:

$$\begin{aligned} midpoint &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{-2 + 3}{2}, \frac{1 + 2}{2}\right) \\ &= \left(\frac{1}{2}, \frac{3}{2}\right) \end{aligned}$$

The midpoint is located at $\left(\frac{1}{2}, \frac{3}{2}\right)$.

EXAMPLE 3: A circle has one diameter with endpoints at $A(-2, 3)$ and $B(4, 11)$. Find the coordinates of the center of the circle.

The center is the **midpoint** of a diameter. Let's put $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (4, 11)$ in the midpoint formula:

$$\begin{aligned} midpoint &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{-2 + 4}{2}, \frac{3 + 11}{2}\right) \\ &= \left(\frac{2}{2}, \frac{14}{2}\right) \\ &= (1, 7) \end{aligned}$$

Thus, the center is the point $(1, 7)$.

EXAMPLE 4: A is the point $(a, -5)$, while B is the point $(0, b)$. Find the values of a and b for which the midpoint of AB is at $(1, -3)$.

Put $(x_1, y_1) = (a, -5)$ and $(x_2, y_2) = (0, b)$ in the midpoint formula:

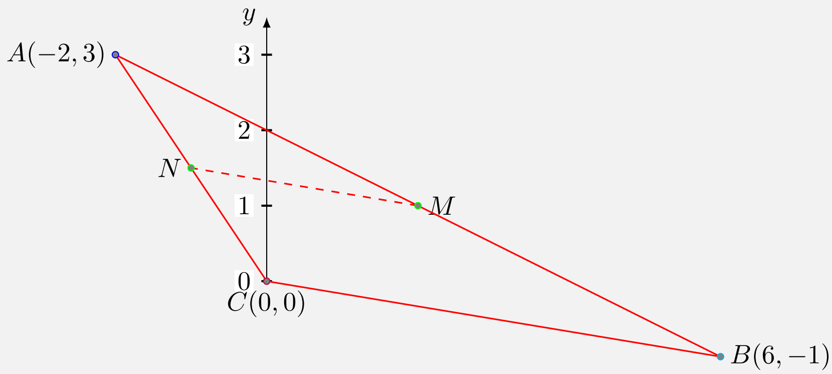
$$\begin{aligned} \textit{midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ \implies (1, -3) &= \left(\frac{a + 0}{2}, \frac{-5 + b}{2}\right) \end{aligned}$$

Since coordinates are ordered pairs, we have to equate component-by-component – that is, the x-components separately, and the y-components separately. Like so:

$$\begin{aligned} \text{first components}: \quad 1 &= \frac{a}{2} \\ \therefore 2 &= a \\ &\ldots\vdots\ldots \\ \text{second components}: \quad -3 &= \frac{-5 + b}{2} \\ -6 &= -5 + b \\ \therefore -1 &= b \end{aligned}$$

Thus, $a = 2$ and $b = -1$.

EXAMPLE 5: Consider $\triangle ABC$ with vertices at $A(-2,3)$, $B(6,-1)$, $C(0,0)$. Let M be the midpoint of AB, and let N be the midpoint of AC. PROVE that MN is *parallel* to BC.



Since M is the midpoint of AB, it is the point $\left(\frac{-2+6}{2}, \frac{3+-1}{2}\right) = (2,1)$. Similarly, N being the midpoint of AC means that it is the point $\left(\frac{-2+0}{2}, \frac{3+0}{2}\right) = \left(-1, \frac{3}{2}\right)$.

✍ MN is parallel to BC: we use **slopes**.

$$\begin{aligned} \text{slope of } MN &= \frac{3/2-1}{-1-2} \\ &= \frac{1/2}{-3} \\ &= -\frac{1}{6} \\ \text{slope of } BC &= \frac{-1-0}{6-0} \\ &= -\frac{1}{6} \\ &\therefore MN \text{ is parallel to } BC \end{aligned}$$

✍ (ASIDE): if you use the length formula from the previous lesson, you will notice that length of MN is half the length of BC. In general, a line which connects the midpoints of two sides of a triangle is parallel to the third side and equal to half its length. We may come across this again in our lesson on properties of triangles.

Finally, coffee time ···