LESSON 5: Some applications of linear systems

Given a linear system word problem involving two variables:

- ✍ try to understand the question (some key terms like sum, difference, average, etc, may help)
- ✍ choose letters to represent (the two) unknowns in the question
- ✍ formulate two linear equations using the given information
- ✍ solve the linear equations formulated above
- ✍ CHECK $\cdots$

Then $\cdots$

That's right.

EXAMPLE 1: The sum of two numbers is 12, and their difference is 4. Find the two numbers.

Let the two numbers be x and y. Sum means **addition**, while difference means **subtraction**. So:

$$x + y = 12 \tag{1}$$

$$x - y = 4 \tag{2}$$

Let's eliminate y. Then we have to ADD equations (1) and (2) above, due to the opposite signs in the coefficients of y. This gives:

$$2x = 16 \implies x = \frac{16}{2} = 8$$

We can now find the value of y, using any of the original equations. For example, using (1):

$$x + y = 12 \implies 8 + y = 12 \implies y = 12 - 8 \implies y = 4.$$

Therefore, the two numbers are 8 and 4.

CHECKING $\cdots$
$8 + 4 = 12$ and $8 - 4 = 4$.

EXAMPLE 2: Two cell phone companies charge a monthly fee plus an additional cost for each minute of time used. "Talk More" charges a flat fee of $\$30$ plus an additional $\$5$ per hour, while "We Talk" charges a flat fee of $\$50$ plus an additional $\$3$ per hour.

(a) Write down two linear equations to represent the cost of using each of the service providers.

Let x be the number of hours, and let y be the cost.

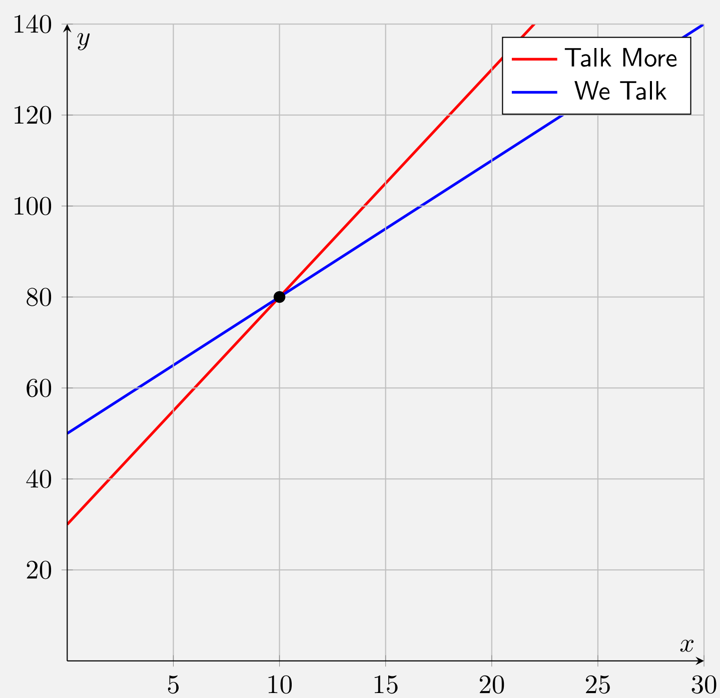
Talk More $y = 5x + 30$

We Talk $y = 3x + 50$

(b) Create a table of values for the following number of hours: $0, 5, 10, 15, \cdots, 30$.

Hours, x	Talk More, $y = 5x + 30$	We Talk, $y = 3x + 50$
0	30	50
5	55	65
10	80	80
15	105	95
20	130	110
25	155	125
30	180	140

(c) Use the table of values in part b) to sketch the graphs of the two lines on the same grid



(d) What is the POI, and what does it mean in relation to the cell phone plans?

From the graph, the point of intersection is $(10, 80)$. In relation to the cell phone plans, it means that at 10 hours of call, the cost of using the two service providers is the same, namely $\$80$.

EXAMPLE 3: A certain store sells 2 pencils and 5 pens for $\$3$, while 6 pencils and 1 pen are sold for $\$2$. Find the cost of a pencil and the cost of a pen.

Let the cost of a pencil be c dollars, and let the cost a pen be n dollars. Then:

$$2c + 5n = 3 \tag{3}$$

$$6c + n = 2 \tag{4}$$

Let's apply **substitution** method. Isolate n from equation (4): $n = 2 - 6c$. Next, replace the n in equation (3) with $2 - 6c$ and continue the algebra:

$$\begin{aligned}
2c + 5n &= 3 \\
2c + 5(2 - 6c) &= 3 \\
2c + 10 - 30c &= 3 \\
-28c &= 3 - 10 \\
-28c &= -7 \\
c &= \frac{-7}{-28} \\
\therefore c &= \frac{1}{4} \\
\text{Or, } c &= 0.25 \\
\cdots &\vdots \cdots \\
\cdots &\vdots \cdots \\
n &= 2 - 6c \\
\implies n &= 2 - 6 \times 0.25 \\
&= 2 - 1.5 \\
\therefore n &= 0.5
\end{aligned}$$

We chose to leave our answers in decimal form this time because we're dealing with money.

That store sells a pencil for 25 cents, and a pen for 50 cents.

It is useful to know the "category" that a given word problem belongs to:

- mixtures
- speed, distance, time
- money
- and others

This helps in the initial stage of formulating appropriate linear system from the given information. As you practise more, you will become accustomed to the process.

EXAMPLE 4: If you have 30 coins made up of nickels and quarters, and the total value of your money is $\$5.5$, how many nickels and how many quarters do you have?

Let the number of nickels be n, and let the number of quarters be q. Since you have 30 coins overall, the first equation is

$$n + q = 30 \tag{5}$$

To obtain the second equation, use the fact that a nickel is worth 5 cents, while a quarter is worth 25 cents. Also, convert the $\$5.5$ to its cents equivalent.

$$5n + 25q = 550 \tag{6}$$

Let's eliminate n. Divide equation (6) by 5 to obtain

$$n + 5q = 110 \tag{7}$$

Now subtract equation (5) from equation (7): $\Longrightarrow$ $4q = 80$, and so $q = 20$. In turn, $n = 10$.

You have 10 nickels and 20 quarters.

EXAMPLE 5: A boat travelled 240 miles downstream in 12 hours. The return journey took 48 hours. Find the speed of the boat in still water and the speed of the current.

Ignoring units, let the speed of the boat in still water be s, and let the speed of the current be c. In going downstream, the boat is supported by the current, so its resultant speed is $s + c$, which must equal $\frac{240}{12}$; on the other hand, the boat goes against the current in the return journey, so its resultant speed then is $s - c$, which must equal $\frac{240}{48}$ (i.e., **speed** equals **distance** divided by **time**). We then have the linear system:

$$s + c = 20 \tag{8}$$

$$s - c = 5 \tag{9}$$

Add equations (8) and (9): $\Longrightarrow$ $2s = 25$, and so $s = 12.5$.

Subtract equation (9) from equation (8): $\Longrightarrow$ $2c = 15$, and so $c = 7.5$.

The speed of the boat in still water is 12.5 miles per hour, while the speed of the current is 7.5 miles per hour.