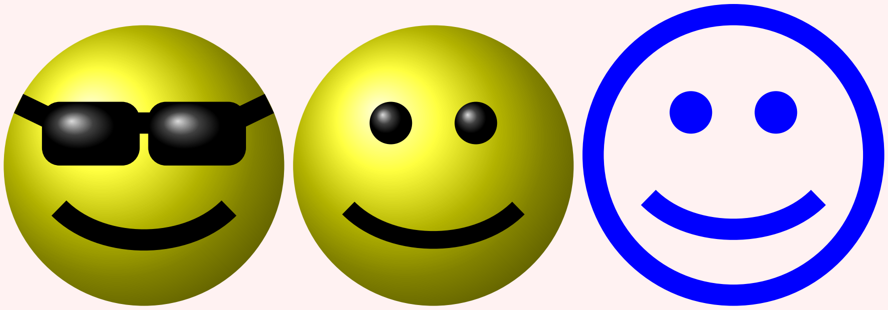
LESSON 4: Solution of linear systems using elimination

PROCEDURE: First step is to SMILE



On a more smiling note, the steps involved are simple:

1 Choose a variable to **eliminate**.

2 Check that the chosen variable has the **same coefficients** in both equations. If not, **force** it. (See EXAMPLE 3.)

3 Cancel (eliminate) the chosen variable by adding or subtracting. Then solve the remaining **one variable** linear equation. Then, then $\cdots$.

Then, smile again.

EXAMPLE 1: Solve the linear system using elimination:

$$x + y = 4 \qquad (1)$$

$$x - y = 2 \qquad (2)$$

The **choice** of which variable to eliminate is easy here: x has a **coefficient** of 1 in both equations, while y has a **coefficient** of 1 in the first equation, and a coefficient of -1 in the second equation.

For now let's eliminate y. Then we have to ADD equations (1) and (2) above, due to the opposite signs in the coefficients of y. This gives:

$$2x = 6 \implies x = \frac{6}{2} = 3$$

We can now find the value of y, using any of the original equations. For example, using (1):

$$x + y = 4 \implies 3 + y = 4 \implies y = 4 - 3 \implies y = 1.$$

Therefore, the solution is: $x = 3, \; y = 1$.

EXAMPLE 2: Solve the linear system using elimination:

$$3m + 4n = 7 \quad (3)$$

$$-3m + n = -2 \quad (4)$$

The **choice** of which variable to **eliminate** is again easy: it is m. Of course, nothing stops one from deciding to eliminate n instead, but that requires an extra step.

Since m has **opposite coefficients** in equation (3) and equation (4), we ADD both equations to eliminate it. The steps are shown below:

$$\begin{aligned} 5n &= 5 \\ \implies n &= 1 \\ &\vdots \ldots \vdots \\ 3m + 4n &= 7 \\ 3m + 4(1) &= 7 \\ 3m + 4 &= 7 \\ 3m &= 7 - 4 \\ 3m &= 3 \\ \implies m &= 1 \end{aligned}$$

Therefore, the solution is $m = 1, \ n = 1$.

Sometimes, the coefficients of the variables may have to be recast before elimination can proceed.

See the next example.

EXAMPLE 3: Use the method of elimination to solve the linear system:

$$7x + 2y = 10 \quad (5)$$

$$3x + 8y = 11 \quad (6)$$

First, we **choose** to eliminate y. However, being that y **does not have the same coefficients** in both equations, we have to **force** this to be the case. Multiply the first equation, i.e., equation (5), by 4 to obtain $28x + 8y = 40$, an equation that is **equivalent** to (5). Now combine this with equation (6):

$$28x + 8y = 40$$

$$3x + 8y = 11$$

SUBTRACT to eliminate y, since the coefficients have the same signs (8 in both cases):

$$25x = 29 \implies x = \frac{29}{25}$$

Substitute $x = \frac{29}{25}$ in either (5) or (6) to obtain the value of y. Let's use (5).

$$\begin{aligned} 7x + 2y &= 10 \\ \implies 7\left(\frac{29}{25}\right) + 2y &= 10 \\ \implies \frac{203}{25} + 2y &= 10 \\ 2y &= 10 - \frac{203}{25} \\ 2y &= \frac{47}{25} \\ \therefore y &= \frac{47}{50} \end{aligned}$$

The solution is $x = \frac{29}{25}, \ y = \frac{47}{50}$. Let's **VERIFY**:

In (5), we have $7x + 2y = 10$. Consider the **left side**, namely $7x + 2y$. Using our x and y values, this becomes

$$7\left(\frac{29}{25}\right) + 2\left(\frac{47}{50}\right) = \frac{203}{25} + \frac{47}{25} = \frac{250}{25} = 10,$$

which equals the **right side**. Now, in (6) we have $3x + 8y = 11$. The **left side** of this is $3x + 8y$. Substituting our x and y values again gives

$$3\left(\frac{29}{25}\right) + 8\left(\frac{47}{50}\right) = \frac{87}{25} + \frac{188}{25} = \frac{275}{25} = 11,$$

which equals the **right side**. Thus the solution is **verified**.

EXAMPLE 4: Solve the linear system using elimination method:

$$3x - 7y = 6 \tag{7}$$

$$5x + 8y = 10 \tag{8}$$

Let's eliminate x. Multiply equation (7) by 5, and then multiply equation (8) by 3. This way, x will now have the same coefficient (of 15) in the resulting equivalent equations:

$$\text{equation } (7) \times 5 \implies 15x - 35y = 30$$
$$\text{equation } (8) \times 3 \implies 15x + 24y = 30$$

Now, SUBTRACT to eliminate x: $59y = 0 \implies y = 0$. It then remains to get x. Use the value of y in any of the original equations, for example, equation (7):

$$3x - 7y = 6 \implies 3x - 7(0) = 6 \implies 3x = 6 \implies x = 2.$$

The solution to the linear system is $x = 2,\ y = 0$.

EXAMPLE 5: Solve the linear system using elimination method:

$$x + 8y = 9 \tag{9}$$

$$3x + 4y = -3 \tag{10}$$

Let's eliminate x. Multiply equation (9) by 3, and leave equation (10) as it is (or just multiply it by 1).

$$\text{equation } (9) \times 3 \implies 3x + 24y = 27$$
$$\text{equation } (10) \times 1 \implies 3x + 4y = -3$$

Now, SUBTRACT: $20y = 30 \implies y = \frac{3}{2}$. It then remains to get x. Use this value of y in any of the original equations, for example, equation (9):

$$x + 8y = 9 \implies x + 8\left(\frac{3}{2}\right) = 9 \implies x + 12 = 9 \implies x = -3.$$

The solution to the linear system is $x = -3,\ y = \frac{3}{2}$. CHECKING $\cdots$

$$\begin{aligned} \text{In equation } (9):\ \ x + 8y &= 9 \\ -3 + 8\left(\frac{3}{2}\right) &= -3 + 12 \\ &= 9 \\ \text{In equation } (10):\ \ 3x + 4y &= -3 \\ 3(-3) + 4\left(\frac{3}{2}\right) &= -9 + 6 \\ &= -3 \end{aligned}$$

CONFIRMED.