LESSON 3: Solution of linear systems using substitution

PROCEDURE: Easier DONE than SAID

Once you know how to **isolate variables**, the process should be a talk in the park. Or, a walk in the park.

1 if one of the two linear equations is already written in the form $y = mx + b$, then replace/substitute $mx + b$ for y in the other equation.

2 if none of the two linear equations is given in the form $y = mx + b$, then first **isolate a variable** using an equation that's convenient.

Easy-peasy.

EXAMPLE 1: Solve the linear system by substitution:

$$y = 2x + 3 \quad (1)$$

$$y = -5x - 4 \quad (2)$$

Both equations have been put in convenient forms. Simply equate the right sides of both equations:

$$\begin{aligned} 2x + 3 &= -5x - 4 \\ 2x + 5x &= -4 - 3 \\ 7x &= -7 \\ \therefore x &= \text{-}1 \\ &\vdots \ldots \vdots \\ y &= 2x + 3 \\ y &= 2(-1) + 3 \\ y &= -2 + 3 \\ \therefore y &= 1 \end{aligned}$$

The solution is $x = \text{-}1, y = 1$. Next, CHECK, and SMILE!!!

EXAMPLE 2: **Solve the linear system by the substitution method**:

$$y = 3x - 5 \qquad (3)$$

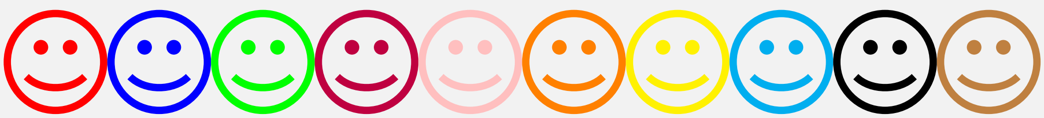
$$y = x + 3 \qquad (4)$$

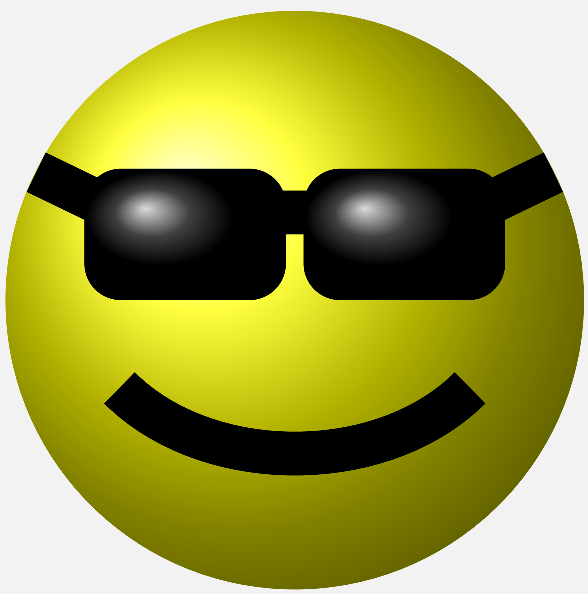
$$\begin{aligned} 3x - 5 &= x + 3 \\ 3x - x &= 3 + 5 \\ 2x &= 8 \\ \therefore x &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

$$\vdots \ldots \vdots$$

$$\begin{aligned} y &= 3x - 5 \\ y &= 3(4) - 5 \\ y &= 12 - 5 \\ \therefore y &= 7 \end{aligned}$$

Checking the solutions: $x = 4, \; y = 7$
FIRST EQUATION TO CHECK: $y = 3x - 5 \implies y = 3 \times 4 - 5 = 12 - 5 = 7$. **Fine.**
SECOND EQUATION TO CHECK: $y = x + 3 \implies y = 4 + 3 = 7$. **Works. Fine.**





EXAMPLE 3: Solve the linear system using substitution method:

$$y = 4x + 6 \tag{5}$$

$$y = -2x + 2 \tag{6}$$

$$\begin{aligned}
4x + 6 &= -2x + 2\\
4x + 2x &= 2 - 6\\
6x &= -4\\
x &= -\frac{4}{6} \quad \textbf{or} \quad x = -\frac{2}{3}\\
&\vdots \cdots \vdots\\
y &= 4x + 6\\
y &= 4\left(-\frac{2}{3}\right) + 6\\
&= -\frac{8}{3} + 6\\
&= -\frac{8}{3} + \frac{18}{3}\\
y &= \frac{10}{3}
\end{aligned}$$

$\therefore x = -\frac{2}{3},\ y = \frac{10}{3}$. **Next, CHECK and SMILE!!!**

EXAMPLE 4: Solve the linear system using substitution method:

$$2x + 3y = 21 \tag{7}$$

$$x - 5y = -22 \tag{8}$$

We need to first **isolate** one of the two variables, using any of the two equations. Since it is easier to isolate x from the second equation, we make use of it. From (8), we have: $x = 5y - 22$. Substitute $5y - 22$ for x in equation (7):

$$\begin{aligned} 2(5y - 22) + 3y &= 21 \\ 10y - 44 + 3y &= 21 \\ 13y &= 65 \\ \therefore y &= 5 \end{aligned}$$

Next, we use the fact that $x = 5y - 22$ to obtain the value of x:

$$\begin{aligned} x &= 5y - 22 \\ &= 5(5) - 22 \\ &= 3 \end{aligned}$$

The solution to the linear system is $x = 3, y = 5$. DON'T FORGET TO CHECK.

EXAMPLE 5: Solve the linear system using substitution method:

$$5x - 6y = -2 \tag{9}$$

$$3x + 4y = \frac{13}{5} \tag{10}$$

Isolate x from the first equation. From (9), we have: $x = \frac{6y-2}{5}$. Substitute $\frac{6y-2}{5}$ for x in equation (10):

$$\begin{aligned} 3\left(\frac{6y-2}{5}\right) + 4y &= \frac{13}{5} \\ 5 \times 3\left(\frac{6y-2}{5}\right) + 5 \times 4y &= 5 \times \frac{13}{5} \quad \textbf{CLEAR FRACTIONS} \\ 3(6y-2) + 20y &= 13 \\ 18y - 6 + 20y &= 13 \\ 38y &= 19 \\ \therefore y &= \frac{1}{2} \end{aligned}$$

Next, we use the fact that $x = \frac{6y-2}{5}$ to obtain the value of x. We get $x = \frac{6 \times \frac{1}{2} - 2}{5} = \frac{3-2}{5} = \frac{1}{5}$. The solution to the linear system is $x = \frac{1}{5}, y = \frac{1}{2}$.

CHECK. In (9): LEFT side $= 5x - 6y = 5 \times \frac{1}{5} - 6 \times \frac{1}{2} = 1 - 3 = -2 =$ RIGHT side. Same (10).