LESSON 2: Graphical solution of linear systems

Linear system: two or more linear equations treated simultaneously. Today's lesson focuses on only two.

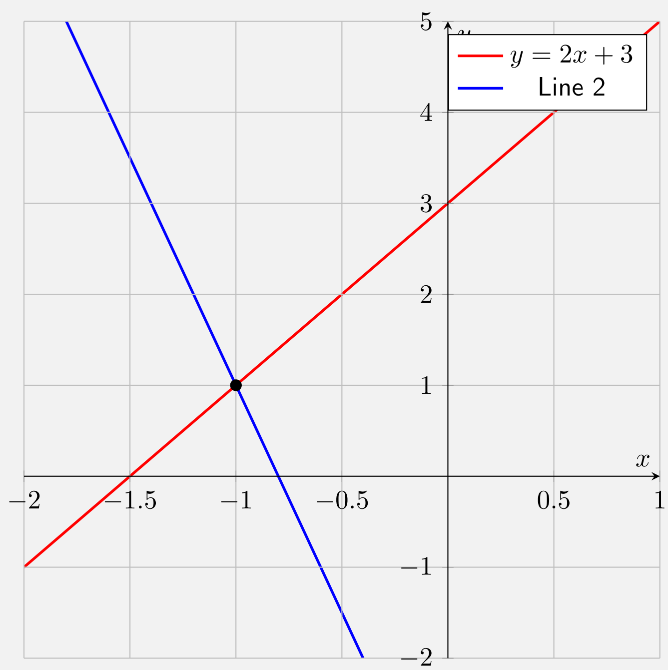
Three **possibilities**:

ONE point of intersection; NO point of intersection (parallel lines); INFINITELY many points of intersection (coincident lines).

EXAMPLE 1: Solve the linear system using graphical method

$$y = 2x + 3 \tag{1}$$

$$y = -5x - 4 \tag{2}$$



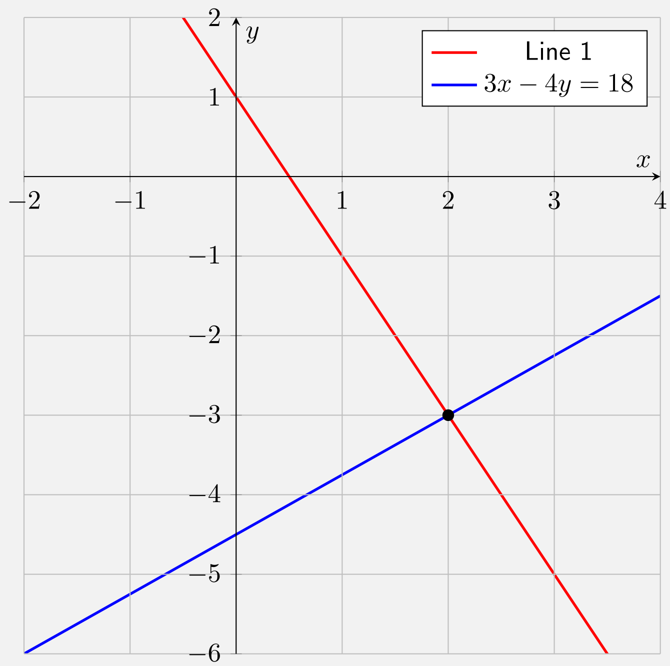
Thus, the Point Of Intersection (POI) is $(-1, 1)$.

CHECKING THE SOLUTIONS:
Using $y = 2x + 3$, we have $y = 2(-1) + 3 = -2 + 3 = 1$.
Using $y = -5x - 4$, we have $y = -5(-1) - 4 = 5 - 4 = 1$. Both equations give the same correct value of y.

EXAMPLE 2: Solve the linear system using graphical method

$$2x + y = 1 \tag{3}$$

$$3x - 4y = 18 \tag{4}$$



From the graph, the POI is $(2, -3)$. This is the solution to the given linear system.

CHECKING THE SOLUTION:

$$\begin{aligned} 2x + y &= 1 \\ 2(2) + (-3) &= 4 - 3 \\ &= 1 \end{aligned}$$

$$\cdots\vdots\cdots$$

$$\begin{aligned} 3x - 4y &= 18 \\ 3(2) - 4(-3) &= 6 + 12 \\ &= 18 \end{aligned}$$

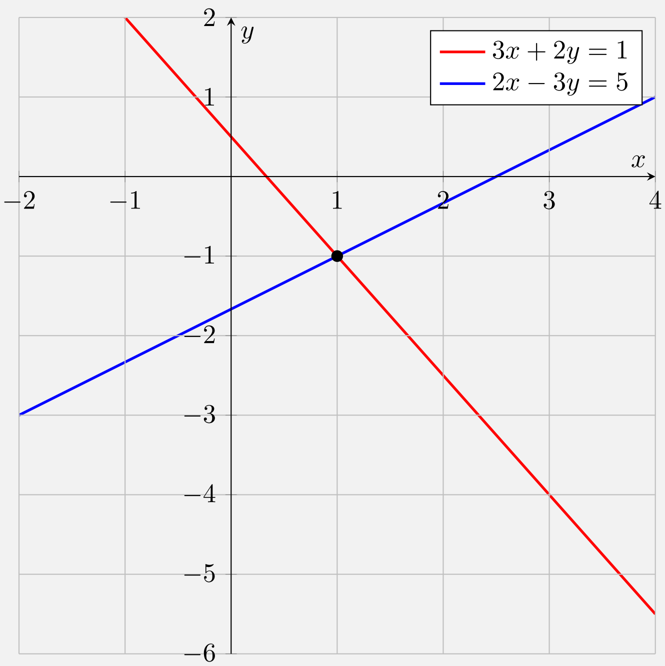
Unrelated info

For something totally unrelated to today's lesson: Line 1 and Line 2 (labels used in the two preceding examples) are just reminders of our city's subway system – CLICK HERE to go there.

EXAMPLE 3: Solve the linear system using graphical method

$$3x + 2y = 1 \tag{5}$$

$$2x - 3y = 5 \tag{6}$$



From the graph, the POI is $(1, -1)$. This is the solution to the given linear system.

CHECKING THE SOLUTION:

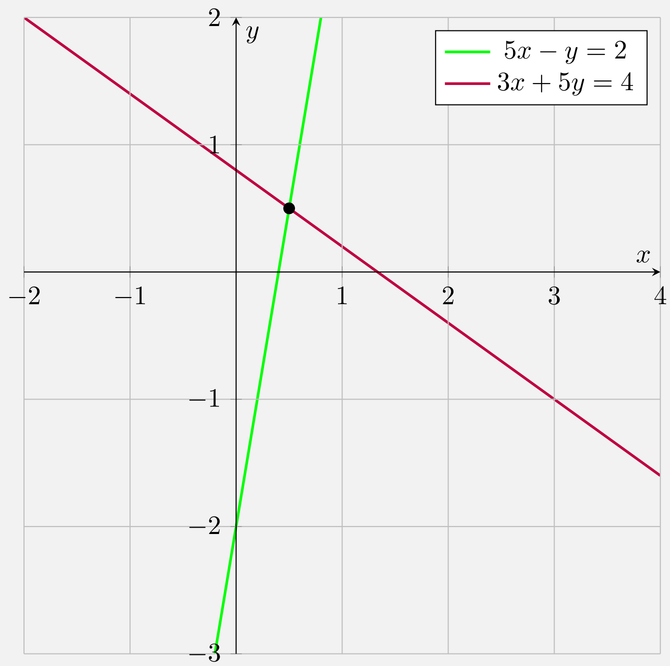
$$\begin{aligned} 3x + 2y &= 1 \\ 3(1) + 2(-1) &= 3 - 2 \\ &= 1 \\ 2x - 3y &= 5 \\ 2(1) - 3(-1) &= 2 + 3 \\ &= 5 \end{aligned}$$

CONCERNING THE GRAPHS:
Consider equation (5) above, namely $3x + 2y = 1$. The x-intercept is $\frac{1}{3} = 0.3333\cdots$, which may not be convenient to locate on a grid. For this reason, sketching this graph using x and y intercepts is not the best. One way out is to re-write the equation in **slope-y-intercept form**: $y = -\frac{3}{2}x + \frac{1}{2}$. Then, start at $(0, \frac{1}{2})$, move **down** 3 units, and move **right** 2 units. Or, move **up** 3 units, and move **left** 2 units. Continue this process until several points have been captured and connected.
Better still, use **guess-and-check** to obtain two (or more) points on the line $3x + 2y = 1$. For example, put $x = -1$ and solve for y: $3(-1) + 2y = 1 \implies y = 2$. So one point is $(-1, \mathbf{2})$. ETC.

EXAMPLE 4: Solve the linear system using graphical method

$$5x - y = 2 \tag{7}$$

$$3x + 5y = 4 \tag{8}$$



From the graph, the POI appears to be $\left(\frac{1}{2}, \frac{1}{2}\right)$. This needs to be VERIFIED.

$$\begin{aligned} 5x - y &= 2 \\ 5\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right) &= \frac{4}{2} \\ &= 2 \\ \cdots &\vdots \cdots \\ 3x + 5y &= 4 \\ 3\left(\frac{1}{2}\right) + 5\left(\frac{1}{2}\right) &= \frac{8}{2} \\ &= 4. \end{aligned}$$

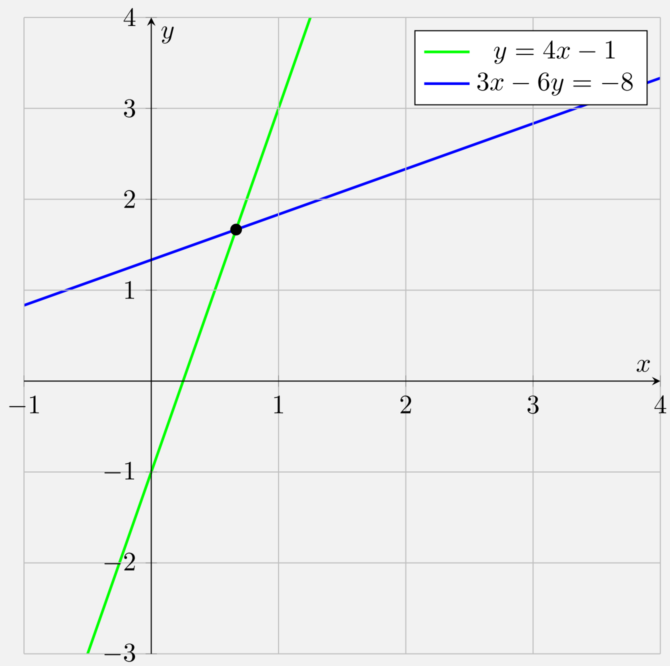
Unfamiliar fractions or decimals as POI

NOTE that it is sometimes challenging to read the coordinates of the point of intersection from a graph. This is one major limitation of the graphical method of solving linear systems. For this reason, other methods of solving linear systems will be considered.

EXAMPLE 5: Solve the linear system using graphical method

$$y = 4x - 1 \tag{9}$$

$$3x - 6y = -8 \tag{10}$$



The comment at the end of EXAMPLE 4 is even more relevant here. How do we determine the **exact** POI from the graph above? Another method of solution is needed. For now, take the POI as $\left(\frac{2}{3}, \frac{5}{3}\right)$.

CHECKING THE SOLUTION:

$$\begin{aligned} y &= 4x - 1 \\ \textbf{Left side} &= y \\ &= \frac{5}{3} \\ \textbf{Right side} &= 4x - 1 \\ &= 4\left(\frac{2}{3}\right) - 1 \\ &= \frac{8}{3} - 1 \\ &= \frac{5}{3} \\ \therefore \textbf{Left side} &= \textbf{Right side} \\ &\cdots\vdots\cdots \\ 3\left(\frac{2}{3}\right) - 6\left(\frac{5}{3}\right) &= 2 - 10 \\ &= -8 \end{aligned}$$

CONFIRMED!!!