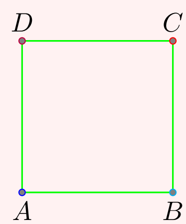
LESSON 11: Properties of quadrilaterals

Beautiful analytic geometry continues ···

✍ a quadrilateral is a closed shape with four straight sides. E.g. squares, rectangles, rhombuses, etc.

✍ in a square:

- **all sides are equal in length**;
- **opposite sides are parallel**;
- **adjacent sides meet at right angles**.



✍ in a rectangle:

- **opposite sides are equal in length**;
- **opposite sides are parallel**;
- **adjacent sides meet at right angles**.

✍ in a rhombus:

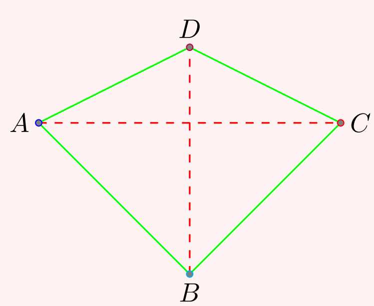
- **all sides are equal**;
- **opposite sides are parallel**;
- **adjacent sides do not necessarily meet at right angles**.

✍ in a parallelogram:

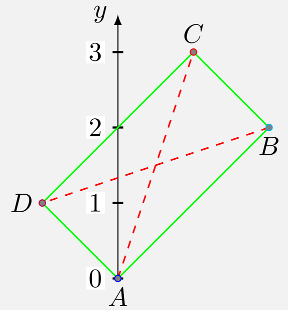
- **opposite sides are equal**;
- **opposite sides are parallel**;
- **adjacent sides do not necessarily meet at right angles**.

✍ in a kite:

- **two pairs of adjacent sides are equal in length** (e.g. $AD = DC$ and $AB = BC$ below):



EXAMPLE 1: What type of quadrilateral is formed by the points $A(0,0)$,$B(2,2)$,C(1,3),$D(-1,1)$?



- the diagram appears to be that of a rectangle, but this needs verification.
- opposite sides are parallel and adjacent sides meet at right angles:
  - AB is parallel to DC:

$$\textbf{slope of } AB = \frac{2-0}{2-0} = 1 \quad \textbf{and slope of } DC = \frac{3-1}{1--1} = 1$$

  - BC is parallel to AD:

$$\textbf{slope of } BC = \frac{3-1}{1-2} = -1 \quad \textbf{and slope of } AD = \frac{1-0}{-1-0} = -1$$

  - $AB \perp BC$: **because the slope of AB is 1, while the slope of BC is -1**.
  - $AD \perp DC$: **because the slope of AD is -1, while the slope of DC is 1**.
- opposite sides are equal in length:
  - $AB = DC$:

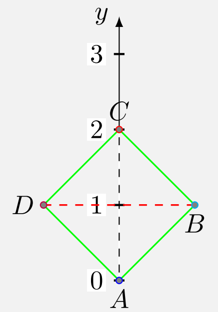
$$\begin{aligned} AB &= \sqrt{(2-0)^2 + (2-0)^2} \\ &= \sqrt{8} \\ CD &= \sqrt{(-1-1)^2 + (1-3)^2} \\ &= \sqrt{8} \end{aligned}$$

  - $BC = AD$:

$$\begin{aligned} BC &= \sqrt{(1-2)^2 + (3-2)^2} \\ &= \sqrt{2} \\ AD &= \sqrt{(-1-0)^2 + (1-0)^2} \\ &= \sqrt{2} \end{aligned}$$

Confirmed!!! The quadrilateral $ABCD$ is a rectangle. The question has been rectified.

EXAMPLE 2: What type of quadrilateral is formed by the points $A(0,0)$,$B(1,1)$,C(0,2),$D(-1,1)$?



Looks like a square or a rhombus, but we need to check. Slopes first:

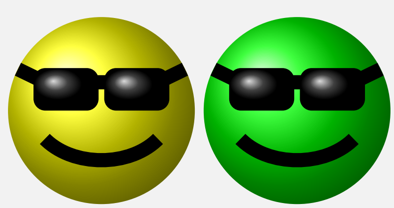
- slope of AB : $m_{AB} = 1$
- slope of BC : $m_{BC} = -1$
- slope of CD : $m_{CD} = 1$
- slope of DA : $m_{DA} = -1$

Thus, $AB \perp BC, BC \perp CD,\ CD \perp DA,\ DA \perp AB$. Since adjacent sides are perpendicular, this rules out the rhombus possibility. Next, we use lengths to confirm that it is a square:

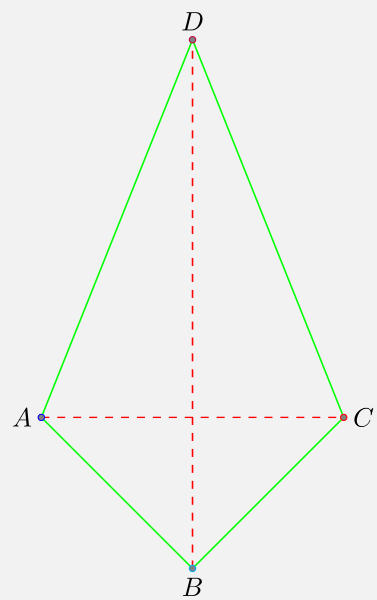
$$\begin{aligned} AB &= \sqrt{(1-0)^2+(1-0)^2} \\ &= \sqrt{2} \\ BC &= \sqrt{(0-1)^2+(2-1)^2} \\ &= \sqrt{2} \\ CD &= \sqrt{(-1-0)^2+(1-2)^2} \\ &= \sqrt{2} \\ DA &= \sqrt{(0--1)^2+(0-1)^2} \\ &= \sqrt{2} \end{aligned}$$

We obtain a square.

One more example, but before then:



EXAMPLE 3: What type of quadrilateral is formed by the points $A(0,0)$,$B(2,-2)$,C(4,0),$D(2,5)$?



Looks like a kite, but never rely on lookalikes – especially in math. To verify, we check that two adjacent sides are equal in lengths.

$$
\begin{aligned}
AB &= \sqrt{(2-0)^2 + (-2-0)^2} \\
&= \sqrt{8} \\
BC &= \sqrt{(4-2)^2 + (0--2)^2} \\
&= \sqrt{8} \\
CD &= \sqrt{(2-4)^2 + (5-0)^2} \\
&= \sqrt{29} \\
DA &= \sqrt{(0-2)^2 + (0-5)^2} \\
&= \sqrt{29}
\end{aligned}
$$

We obtain a kite, since the adjacent sides $AB\&BC$ are equal in length, and the adjacent sides $CD\&DA$ are also equal in length.

Now: